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TRANSIENT RESPONSE ANALYSIS OF LARGE-SCALE ROTOR-BEARING SYSTEM WITH STRONG NON-LINEAR ELEMENTS BY A TRANSFER MATRIX-NEWMARK FORMULATION ITEGRATION METHOD

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This paper extends the transfer matrix technique (TMT) to the transient response analysis of a large complex non-linear rotor-bearing system by a transfer matrix-Newmark formulation itegration method. Firstly, the transfer matrix is obtained via the Newmark formulation. Secondly, the deflections and velocities at the stations, containing non-linear element, are determined by iteration. Finally, the deflections, velocities and accelerations of all stations are computed by TMT and the Newmark formulation consistent with the boundary conditions. In order to eliminate the numerical instability of TMT, the transfer vector $\{f^T \vdots \ddot{e}^T\}^T$ is used, instead of the traditional one $\{f^T \vdots e^T\}^T$. Owing to the advantages of TMT and the Newmark formulation, this method can be applied to calculate the transient response of a large-scale rotor-bearing system with strong non-linear elements, and to analyze its stability. Two illustration examples are given, and the results agree well with those by Runge–Kutta method, and by modal synthetic method.

1. INTRODUCTION

During studying the dynamic character of a rotor-bearing system, the transfer matrix technique is often used to calculate the critical speeds, modes, steady state unbalance response, and analyze its stability because of that the effects of various factors can be considered; the minimum number of variables and equations are required; and the method is easy to apply yet with satisfactory accuracy [1–6]. For example, in 1982, Chu and Pilkey first used the Riccatitransfer matrix technique in the transient response analysis of linear rotor-bearing systems [1]. In 1983, a transfer matrix-direct integration method was used to calculate the critical speeds, modes, steady state unbalance response of a linear rotor-bearing system by Gu Jialiu [2]. In 1986, an improved transfer matrix-direct integration method was used to obtain the steady state unbalance response of a weak non-linear rotor-bearing system, and to analyze its stability by Gu [3]. An equivalent linearization-Prohl transfer matrix iteration method was proposed by Gu and Chen to compute the steady state unbalance response of complex non-linear rotor-bearing system [4]. However, the

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transient responses and their stability analysis of a non-linear rotor-bearing system are usually studied by the modal synthetic method [5] and the finite element method [6].

This paper extends TMT to the transient response analysis of non-linear rotor-bearing systems by a transfer matrix-Newmark formulation integration method. With the advantages of transfer matrix method and improved numerical stability, this method is applicable to either weak or strong non-linear rotor-bearing systems.

2. ESTABLISHING TRANSFER RELATIONSHIP OF DIFFERENT ELEMENTS WITH THE AID OF THE NEWMARK FORMULATION

When a non-linear rotor-bearing system takes part in transient-state motion, its response cannot be expressed as: $\{e\} = \{A\}e^{\lambda t}$. In order to calculate the transient response by TMT, we adopt the Newmark formulation to derive the transfer matrix. The Newmark formulation can be expressed as follows [1]:

$$x_{n+1} = x_n + \Delta t\dot{x}_n + (0{\cdot}5 - \beta)\Delta t^2\ddot{x}_n + \beta\Delta t^2\ddot{x}_{n+1}, \tag{1}$$

$$\dot{x}_{n+1} = \dot{x}_n + (1-\gamma)\Delta t\ddot{x}_n + \gamma\Delta t\ddot{x}_{n+1}.$$

equation (1) can be rewritten in the form

$$\ddot{x}_{n+1} = \frac{1}{\beta\Delta t^2}x_{n+1} - P_n,$$
$$\dot{x}_{n+1} = \frac{\gamma}{\beta\Delta t}x_{n+1} + Q_n \tag{2}$$

or

$$x_{n+1} = \beta\Delta t^2\ddot{x}_{n+1} + R_n,$$
$$\dot{x}_{n+1} = \gamma\Delta t\ddot{x}_{n+1} + \lambda_n, \tag{3}$$

where

$$P_n = \frac{1}{\beta\Delta t^2}x_n + \frac{1}{\beta\Delta t}\dot{x}_n + \left(\frac{1}{2\beta} - 1\right)\ddot{x}_n,$$

$$Q_n = -\frac{\gamma}{\beta\Delta t}x_n + \left(1 - \frac{\gamma}{\beta}\right)\dot{x}_n + \Delta t\left(1 - \frac{\gamma}{2\beta}\right)\ddot{x}_n, \tag{4}$$

$$R_n = x_n + \Delta t\dot{x}_n + (0{\cdot}5 - \beta)\Delta t^2\ddot{x}_n,$$
$$\lambda_n = \dot{x}_n + (1-\gamma)\Delta t\ddot{x}_n. \tag{5}$$

2.1. TRANSFER MATRIX FOR A DISK

According to D'Alembert principles, we can obtain the governing differential equations for a disk as [7]

$$M^r - M^l - J_d\ddot{\theta} - J_p\omega\dot{\delta} = 0,$$

$$U^r - U^l + M\ddot{y} + C_D\dot{y} - me_\mu\omega^2\cos\omega t - mg = 0, \tag{6}$$

$$N^r - N^l - J_d\ddot{\delta} + J_p\omega\dot{\theta} = 0,$$

$$V^r - V^l + m\ddot{z} + C_D\dot{z} - me_\mu\omega^2\sin\omega t = 0.$$

In order to improve the numerical stability of TMT, equation (3), instead of equation (2), is substituted into equation (6) at time $(n+1)\Delta t$. We have

$$\begin{Bmatrix} M \\ U \\ N \\ V \\ \cdots \\ \ddot{\theta} \\ \ddot{y} \\ \ddot{\delta} \\ \ddot{z} \end{Bmatrix}^{r}_{n+1} = \left[\begin{array}{cccc:cccc} 1 & 0 & 0 & 0 & J_d & 0 & J_p\omega\gamma\Delta t & 0 \\ 0 & 1 & 0 & 0 & 0 & -(m+C_D\gamma\Delta t) & 0 & 0 \\ 0 & 0 & 1 & 0 & J_p\omega\gamma\Delta t & 0 & J_d & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -(m+C_D\gamma\Delta t) \\ \hdashline 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{Bmatrix} M \\ U \\ N \\ V \\ \cdots \\ \ddot{\theta} \\ \ddot{y} \\ \ddot{\delta} \\ \ddot{z} \end{Bmatrix}^{l}_{n+1} + \begin{Bmatrix} J_p\omega\lambda_n(\delta) \\ -C_D\lambda_n(y) + me_\mu\omega^2\cos\omega(n+1)\Delta t + mg \\ -J_p\omega\lambda_n(\theta) \\ -C_D\lambda_n(z) + me_\mu\omega^2\sin\omega(n+1)\Delta t \\ \cdots\cdots \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}, \tag{7}$$

where M,N,θ and δ are bending moments and angular displacements in xoy and xoz planes; U, V,y and z are shearing forces and deflections in y and z directions; C_D is the friction coefficient; e_μ is the eccentricity; and ω is the rotating speed.

2.2. TRANSFER MATRIX FOR A SPRING PARTICLE

According to Newton's law, we can obtain the governing differential equations of motion for a spring particle as

$$\begin{aligned} U^r &= U^l - m\ddot{y} - ky + mg, \\ V^r &= V^l - m\ddot{z} - kz. \end{aligned} \tag{8}$$

Substituting equation (3) into equation (8) at time $(n+1)\Delta t$, we obtain

$$\begin{Bmatrix} M \\ U \\ \cdots \\ \ddot{\theta} \\ \ddot{y} \end{Bmatrix}^{r}_{n+1} = \left[\begin{array}{cc:cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -(m+k\beta\Delta t^2) \\ \hdashline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \begin{Bmatrix} M \\ U \\ \cdots \\ \ddot{\theta} \\ \ddot{y} \end{Bmatrix}^{l}_{n+1} + \begin{Bmatrix} 0 \\ mg - kR_n(y) \\ \cdots\cdots \\ 0 \\ 0 \end{Bmatrix}, \tag{9}$$

where k is a spring constant. The transfer matrix in the z direction is similar to equation (9).

2.3. TRANSFER MATRIX FOR A SQUEEZE-FILM DAMPER BEARING

The oil film forces of a squeeze-film damper bearing (Figure 1) are complex non-linear functions of both the deflections and speeds of the bearing centre [8], that is $F_y = F_y(y, z, \dot{y}, \dot{z}), F_z = F_z(y, z, \dot{y}, \dot{z})$. From the dynamic equilibrium conditions of the bearing, we can derive that

$$\begin{aligned} U^r &= U^l - m\ddot{y} + F_y(y, z, \dot{y}, \dot{z}) + mg, \\ V^r &= V^l - m\ddot{z} + F_z(y, z, \dot{y}, \dot{z}). \end{aligned} \tag{10}$$

As the governing differential equations of motion contain the non-linear terms of the deflections and the speeds, the transfer matrix for the bearing deflections cannot be obtained directly, even though the Newmark formulation is used. Therefore, in this station we choose accelerations as transfer variables instead of the deflections. The deflections and speeds are regarded as unknown quantities to be determined. Thus, we can derive that

$$\begin{Bmatrix} M \\ U \\ \cdots \\ \ddot{\theta} \\ \ddot{y} \end{Bmatrix}^r_{n+1} = \begin{bmatrix} 1 & 0 & \vdots & 0 & 0 \\ 0 & 1 & \vdots & 0 & -m \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \vdots & 1 & 0 \\ 0 & 0 & \vdots & 0 & 1 \end{bmatrix} \begin{Bmatrix} M \\ U \\ \cdots \\ \ddot{\theta} \\ \ddot{y} \end{Bmatrix}^l_{n+1} + \begin{Bmatrix} 0 \\ F_{y(n+1)}(y, z, \dot{y}, \dot{z}) + mg \\ \cdots \\ 0 \\ 0 \end{Bmatrix}. \tag{11}$$

The transfer matrix in the z direction is similar to equation (11). Of course, the transfer relationship (11) can be used, only if the non-linear terms on state variables are known.

The point transfer matrix for low- and high-pressure shafts at intershaft squeeze-film damper bearing can be expressed in the same way as equation (11), except the force vector of the high-pressure shaft being $\{0, m_{hi}g - F_{y(n+1)}(y, z, \dot{y}, \dot{z}), 0, F_{z(n+1)}(y, z, \dot{y}, \dot{z}), 0, 0, 0, 0\}^{\mathrm{T}}$.

2.4. TRANSFER MATRIX FOR A SHAFT SEGMENT

According to reference [7], we can obtain the transfer relationship for a masssless shaft segment

$$\begin{Bmatrix} M \\ U \\ \cdots \\ \theta \\ y \end{Bmatrix}^r_{n+1} = \begin{bmatrix} 1 & l & \vdots & 0 & 0 \\ 0 & 1 & \vdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \alpha' & \alpha' l/2 & \vdots & 1 & 0 \\ \alpha' l/2 & \alpha' l^2/6 & \vdots & l & 1 \end{bmatrix} \begin{Bmatrix} M \\ U \\ \cdots \\ \theta \\ y \end{Bmatrix}^l_{n+1}, \tag{12}$$

where $\alpha' = 1/(EJ)$, L is the length of a shaft segment, E is the modulas of elasticity, J is the moment of inertia of a shaft section. Substituting equation (3) into equation (12), we can derive that

$$\begin{Bmatrix} M \\ U \\ \cdots \\ \ddot{\theta} \\ \ddot{y} \end{Bmatrix}^r_{n+1} = \begin{bmatrix} 1 & l & \vdots & 0 & 0 \\ 0 & 1 & \vdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \alpha & \alpha l/2 & \vdots & 1 & 0 \\ \alpha l/2 & \alpha l^2/6 & \vdots & l & 1 \end{bmatrix} \begin{Bmatrix} M \\ U \\ \cdots \\ \ddot{\theta} \\ \ddot{y} \end{Bmatrix}^l_{n+1} + \begin{Bmatrix} 0 \\ 0 \\ \cdots \\ P^l_n(\theta) - P^r_n(\theta) \\ lP^l_n(\theta) + P^l_n(y) - P^r_n(y) \end{Bmatrix}, \tag{13}$$

where $\alpha = \alpha'/\beta \Delta t^2$. The transfer matrix of a shaft in the z direction can be written in the same way.

3. COMPUTING THE TRANSIENT RESPONSE OF NON-LINEAR ROTOR-BEARING SYSTEMS WITH TMT

For linear rotor-bearing systems, all transfer matrixes do not contain the non-linear terms, such as $F_{y(n+1)}(y, z, \dot{y}, \dot{z})$ and $F_{z(n+1)}(y, z, \dot{y}, \dot{z})$. By using TMT, the next instant accelerations of every station are computed from the above transfer matrix and the formal instant conditions. Then, the deflections and speeds of every station are obtained through equation (3). Repeating this procedure, any arbitrary instant acceleration, deflection and speed can be obtained.

For non-linear rotor-bearing systems, however, some transfer matrixes, such as equation (11), contain the non-linear terms of state variables. Only if these non-linear terms are determined, the transfer relationship of the non-linear station might be practical. Before computing the next instant accelerations, the state variables at this instant must be determined. We suggest that the deflection and the speed of the non-linear station be at first predicted by Taylor series. Thus, the non-linear term can be determined. Then, the accelerations of every station are calculated by TMT, and the deflections and the speeds are obtained once again by the Newmark formulation. This iterative procedure is repeated until the deviations of deflections and speeds obtained in successive iterations satisfy the given accuracy. Hence, the transient response of a non-linear rotor-bearing system, where non-linear terms are the complex function of dependent state deflection and speed, can be computed as follows:

(1) By Taylor series, the next instantaneous deflections and speeds of non-linear stations are at first predicted from the former instant deflections, speeds and accelerations:

$$\{e\}_{n+1} = \{e\}_n + \Delta t\{\dot{e}\}_n + o(\Delta t^2),$$
$$\{\dot{e}\}_{n+1} = \{\dot{e}_n\} + \Delta t\{\ddot{e}_n\} + o(\Delta t^2). \tag{14}$$

Then, the non-linear terms of dependent state deflections and speeds at the instant can be determined. For example, the squeeze-film force can be obtained from reference [8]. Now, the transfer relationship (11) can be used.

(2) By TMT, the accelerations of the non-linear stations can be solved from the boundary conditions. Then, the deflections and speeds of the stations are calculated from equation (3).

(3) Compare the deflections and speeds obtained in (2) with those obtained in (1). If their deviations do not satisfy the given accuracy, we modify the state values in (1), and repeat the above computation. Thus, a simply iterative procedure is established from (1) to (3). The iterative procedure is repeated until the deviations all satisfy the given accuracy.

(4) By TMT, the instant accelerations of other stations can be calculated. Then, the deflections and speeds of the stations can be determined by equation (3). Thus the instant deflections, speeds and accelerations of all stations can be obtained. The next instant deflections, speeds and accelerations of all stations can be computed in the same way (1)–(4).

Through the above numerical integration, the transient-state and steady state responses of the non-linear system can be obtained. Furthermore, the modes, critical speed and stability of the system can be determined.

4. ANALYZING NUMERICAL STABILITY OF THIS METHOD

With the above method and equation (2), we can obtain the transfer matrix of a spring particle and a disk for the transfer vector $\{f^{\mathrm{T}} \vdots e^{\mathrm{T}}\}^{\mathrm{T}}$:

$$\left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -(m/(\beta \Delta t^2) + k) \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]. \tag{15}$$

Because the numerical values of E and k are very large and the value of Δt is very small, the quantity levels of the lower left elements in equation (12) is much lower than $o(1)$ and the quantity levels of the upper right elements in equations (15) and (16) are much higher than $o(1)$, whereas the levels of the

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \dfrac{J_d}{\beta \Delta t^2} & 0 & \dfrac{J_p \omega \gamma}{\beta \Delta t} & 0 \\ 0 & 1 & 0 & 0 & 0 & -\left(\dfrac{m}{\beta \Delta t^2} + \dfrac{C_D \gamma}{\beta \Delta t}\right) & 0 & 0 \\ 0 & 0 & 1 & 0 & -\dfrac{J_p \omega \gamma}{\beta \Delta t} & 0 & \dfrac{J_d}{\beta \Delta t^2} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -\left(\dfrac{m}{\beta \Delta t^2} + \dfrac{C_D \gamma}{\beta \Delta t}\right) \\ \hline 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \tag{16}$$

other elements of the transfer matrix for transfer vector $\{f^{\mathrm{T}} \vdots e^{\mathrm{T}}\}^{\mathrm{T}}$ are of the same level, $o(1)$. The levels of all elements of the transfer matrix for transfer vector $\{f^{\mathrm{T}} \vdots \ddot{e}^{\mathrm{T}}\}^{\mathrm{T}}$ are the same. Obviously, using the transfer vector $\{f^{\mathrm{T}} \vdots \ddot{e}^{\mathrm{T}}\}^{\mathrm{T}}$ can improve the numerical characteristic of the transfer matrix.

In addition, by using the transfer vector $\{f^{\mathrm{T}} \vdots \ddot{e}^{\mathrm{T}}\}^{\mathrm{T}}$, the acceleration approximation $\{\ddot{e}\}'(\{\ddot{e}\}' = \{\ddot{e}\} + o(\varepsilon))$ is calculated by TMT. Then, the deflection approximation $\{e\}'$ and speed approximation $\{\dot{e}\}'$ are obtained by equation (3). Thus

$$\{\dot{e}\}' = \{\dot{e}\} + o(\varepsilon)\Delta t,$$

$$\{e\}' = \{e\} + o(\varepsilon)\Delta t^2. \tag{17}$$

From equation (17), the errors of the deflection and the speed are greatly decreased. Then, according to the obtained instant state response, the next instant acceleration, deflection and speed are computed.

However, by using the transfer vector $\{f^{\mathrm{T}} \vdots e^{\mathrm{T}}\}$, the deflection approximation $\{e\}'(\{e\}' = \{e\} + o(\varepsilon))$ is calculated by TMT. Then, the speed approximation $\{\dot{e}\}'$ and the acceleration approximation $\{\ddot{e}\}'$ are determined by equation (2). Thus

$$\{\dot{e}\}' = \{\dot{e}\} + o(\varepsilon)\Delta t^{-1},$$
$$\{\ddot{e}\}' = \{\ddot{e}\} + o(\varepsilon)\Delta t^{-2}. \tag{18}$$

From equation (18), the errors of the speed and the acceleration are greatly enlarged. Obviously, the numerical stability of this method is improved by using the transfer vector $\{f^{\mathrm{T}} \vdots \ddot{e}^{\mathrm{T}}\}^{\mathrm{T}}$, whereas the numerical stability of this method is degraded by using the traditional transfer vector $\{f^{\mathrm{T}} \vdots e^{\mathrm{T}}\}^{\mathrm{T}}$.

5. NUMERICAL EXAMPLE

Figure 1 shows a flexible rotor system, which is supported on the squeeze-film damper bearings without centralizing springs, and Figure 2 shows a dual rotor-bearing system with intershaft squeeze-film damper. The parameters of the single rotor-bearing system are given as follows. Bearings: radial clearance $C = 0{\cdot}01703$ cm, length $L = 0{\cdot}5281$ cm, diameter $D = 8{\cdot}645$ cm, viscosity $\mu = 3{\cdot}589 \times 10^{-5}$ N S/cm^2; shaft: $l = 30{\cdot}48$ cm, diameter $d = 3{\cdot}8$ cm, modulus of elastisity $E = 20{\cdot}09 \times 10^6$ N/cm^2, density $\rho = 7{\cdot}85$ g/cm^3, mass $m_B = 1{\cdot}66$ kg; disk: mass $m_D = 5$ kg, friction coefficient $C_D = 578{\cdot}87$ kg/s, eccentricity $e_\mu = 1{\cdot}703 \times 10^{-3}$ cm; rotating speed $\omega = 7248$ r.p.m. The mass of the shaft is distributed among the disk according to 17/35, and among the two bearings according to 18/35.

The parameters of the dual rotor-bearing system are given as follows. Intershaft bearing: radial clearance $C = 0{\cdot}018$ cm, length $L = 2$ cm, diameter $D = 6$ cm, viscosity $\mu = 1{\cdot}764 \times 10^{-6}$ N S/cm^2, mass in low and high shaft $M_{LI} = 1000$ g, $M_{HI} = 1700$ g; shaft:

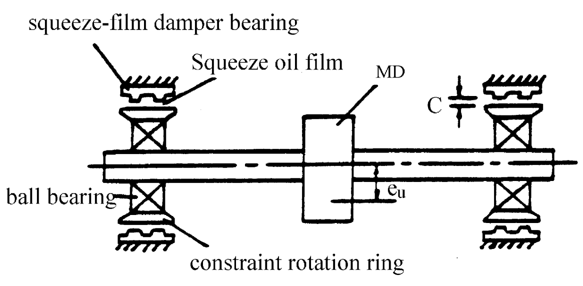

Figure 1. Single rotor-bearing system.

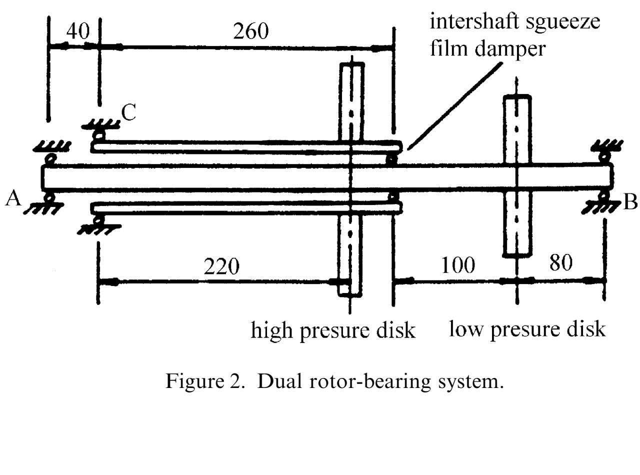

Figure 2. Dual rotor-bearing system.

Table 1

Transient-state value of the disk centre

Time		0·001	0·006	0·011	0·016	0·021
Runge–Kutta method	Y/C	0·200993E+0	0·416018E+0	0·782916E+0	0·594079E+0	0·885948E+0
	Z/C	0·155228E+0	0·903238E−1	0·232300E+0	−0·120969E+0	0·794727E−1
The suggested method	Y/C	0·198315E+0	0·418155E+0	0·782131E+0	0·593329E+0	0·888258E+0
	Z/C	0·153159E+0	0·917701E−1	0·233111E+0	−0·121282E+0	0·779377E−1
Time		0·050	0·055	0·060	0·065	0·070
Runge–Kutta method	Y/C	0·537536E+0	0·960498E+0	0·635011E+0	0·784468E+0	0·830233E+0
	Z/C	0·225652E−1	0·257685E−1	−0·644086E−1	0·102038E+0	−0·942163E−1
The suggested method	Y/C	0·537067E+0	0·961277E+0	0·633797E+0	0·785307E+0	0·829629E+0
	Z/C	0·225681E−1	0·252522E−1	−0·646941E−1	0·101497E+0	−0·947192E−1

low-pressure shaft diameter $d_L = 2\,\text{cm}$, internal and external diameters of high-pressure shaft $d_H = 4\,\text{cm}$, $D_H = 4{\cdot}8\,\text{cm}$, $E = 19{\cdot}6 \times 10^6\,\text{N/cm}^2$, $\rho = 7{\cdot}8\,\text{g/cm}^3$, rotating speed $\omega_L = 3000\,\text{r.p.m.}$, $\omega_H = 5460\,\text{r.p.m.}$; disk: mass of low- and high-pressure disks $M_L = 3293{\cdot}2\,\text{g}$, $M_H = 4242{\cdot}04\,\text{g}$, $J_{PL} = 2J_{dL} = 8{\cdot}065 \times 10^4\,\text{g cm}^2$, $J_{PH} = 2J_{dH} = 1{\cdot}9986 \times 10^5\,\text{g cm}^2$, friction coefficients of low- and high-pressure disks $C_D = 578{\cdot}87\,\text{kg/s}$, eccentricity of high-pressure disk $e_\mu = 3{\cdot}6 \times 10^{-3}\,\text{cm}$; bearing rigidity $K_A = K_B = K_C = 17{\cdot}64 \times 10^{10}\,\text{g/s}^2$. The mass of shafts is distributed among their both ends.

The suggested method and Runge–Kutta method are used to calculate the transient-state and steady state unbalance responses of the disk centre in Figure 1. The transient-state value and steady orbit are shown in Table 1 and Figure 3. The transient solutions by the two methods are almost equal (the deviations are lower than 0·003), and their steady state orbit agree very well.

We also use the suggested method and the modal synthetic method to compute the transient-state responses and steady state orbits of low- and high-pressure shaft centres at intershaft squeez-film damper in Figure 2. The transient response and the steady state orbit are shown in Tables 2 and 3 and Figures 4 and 5, where the dotted line represents the result of the suggested method, and the real line represents the result of modal synthetic method. Although the deviations of two results in Tables 2 and 3 and Figures 4 and 5 are visible, the tendencies agree well. The deviations may be caused by the different way in dispersing shaft segment mass. These results may verify the correctness of the suggested method.

6. CONCLUSIONS

This paper proposes a new analytical and computational method for the transient response of non-linear rotor-bearing systems. In the method, Newmark formulation have two functions: (1) establishing transfer relationship; (2) making numerical integration. Theoretical analysis and computing results all show: by adopting TMT together with the transfer vector $\{ f^{\mathrm{T}} \;\vdots\; \ddot{e}^{\mathrm{T}} \}^{\mathrm{T}}$ and the Newmark formulation, not only the numerical stability of TMT can be improved but also the derivation, programming, and computation are simple and practical. Besides, the computing time and memory space are saved greatly. So, the method is especially applicable to calculating the transient response of large complex non-linear rotor-bearing system, and to analyzing its stability.

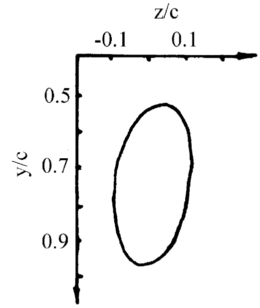

Figure 3. Orbit of the centre of the disk.

TABLE 2

Transient-state value of the high-pressure shaft centre

Time		0·5E−5	0·5E−4	0·1E−3	0·6E−3	0·1E−2
Modal synthetic method	Y_H/C	0·167152E+1	0·206289E+1	0·247358E+1	0·380864E+1	0·344486E−1
	Z_H/C	0·409899E−5	0·195771E−3	−0·299983E−2	0·106717E+0	0·248342E−1
The suggested method	Y_H/C	0·167180E+1	0·195914E+1	0·243108E+1	0·379518E+1	0·442127E+1
	Z_H/C	0·300014E−5	0·392296E−2	−0·228075E−2	0·111489E+0	0·546137E−1
Time		0·6E−2	0·11E−1	0·16E−1	0·21E−1	0·26E−1
Model synthesis method	Y_H/C	0·727916E+0	−0·150425E+1	−0·277063E+0	0·181736E+1	0·515630E+1
	Z_H/C	0·432803E+0	−0·514780E+0	−0·123494E+0	0·549709E+0	0·213258E+1
The suggested method	Y_H/C	0·962474E+0	−0·140162E+1	−0·529249E+0	0·108577E+1	0·500341E+1
	Z_H/C	0·135970E+1	−0·272077E+0	−0·174733E+0	0·592616E+0	0·269895E+1

TABLE 3

Transient-state value of the low-pressure shaft centre

Time		0·5E−5	0·5E−4	0·1E−3	0·6E−3	0·1E−2
Modal synthetic method	Y_L/C	0·145257E+1	0·211392E+1	0·290302E+1	0·325619E+1	0·335866E+1
	Z_L/C	−0·529475E−5	−0·246486E−3	−0·464756E−3	0·113242E+0	0·728010E−1
The suggested method	Y_L/C	0·144939E+1	0·151115E+1	0·241439E+1	0·391290E+1	0·471991E+1
	Z_L/C	0·365309E−5	−0·739748E−3	−0·134332E−2	0·116435E+0	0·141196E+0
Time		0·6E−2	0·11E−1	0·16E−1	0·21E−1	0·26E−1
Model synthetic method	Y_L/C	0·716675E+0	−0·130449E+1	−0·331846E+0	0·151092E+1	0·518432E+1
	Z_L/C	0·424541E+0	−0·551167E+0	−0·123887E+0	0·518579E+0	0·115402E+1
The suggested method	Y_L/C	0·913218E+0	−0·149305E+1	−0·497872E+0	0·110773E+1	0·464771E+1
	Z_L/C	0·116827E+1	−0·402534E+0	−0·147290E+0	0·337479E+0	0·144163E+1

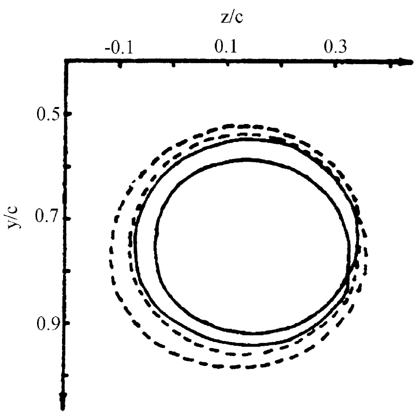

Figure 4. Orbits of the centre of low-pressure shaft.

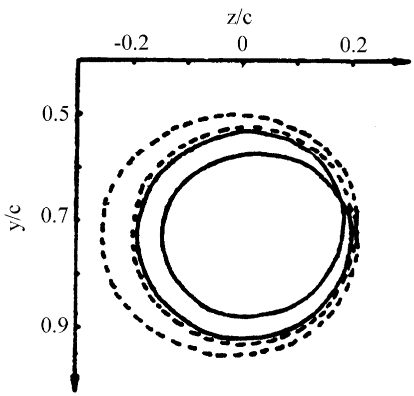

Figure 5. Orbits of the centre of high-pressure shaft.

REFERENCES

1. F. H. Chu and W. D. Pilkey 1982 *Journal of Mechanical Design American Society of Mechanical Engineers* **104**, 384–388. A direct integration technique for the transient analysis of rotating shafts.
2. J. L. Gu 1983 *Acta Aeronautic et Astronautic Sinica* **4**, 48–55. A transfer matrix-direct integration method and its application.
3. J. L. Gu 1986 *Journal of Vibration, Acoustics, Stress, and Reliability in Design, American Society of Mechanical Engineers* **108**, 182–188. An improved transfer matrix-direct integration method for rotor dynamics.
4. Z. P. Gu and S. Q. Chen, 1990 *Chinese Journal of Mechanics and Practice* **12**, 33–36. An equivalent linearization-prohl transfer matrix iteration method.
5. J. M. Gao and S. Q. Chen, 1988 *Journal of Northwestern Polytechnical University* **6**, 343–350. On dynamic boundary conditions in method of component mode synthesis.
6. J. Pandovan *et al.*, 1984 *Computer and Structure* **18**, 629–634. Nonlinear transient finite element analysis of rotor bearing systems.
7. W. T. Thomson 1981 *Theory of Vibration with Application*. Englewood Cliffs, NJ: Prentice-Hall, Inc.
8. S. Q. Chen 1986 *Journal of Northwestern Polytechnical University* **4**, 137–146. Equivalent linearization of a squeeze film damper.

APPENDIX: NOMENCLATURE

$\{e\}$	vector of state variables that are complementary to the $\{f\}$
$\{f\}$	vector of state variables that are homogeneous at the left-hand boundary
$\{\mathrm{J}_d, \mathrm{J}_p\}$	diameter and polar moment of inertia of a disk
H, L	subscript, high-pressure shaft and low-pressure shaft
$n, n+1$	subscript, term at time $n\Delta t$ and $(n+1)\Delta t$
l, r	superscript, left and right
T	superscript, transpose